

Contents

1	Introduction	1
2	Stability of Finite Algorithms	3
2.1	About Algorithms	3
2.2	A Paradoxical Example	4
2.2.1	First Algorithm	4
2.2.2	Second Algorithm	5
2.2.3	Explanation	6
2.3	Accuracy of Elementary Operations	7
2.4	Error Amplification	8
2.4.1	Cancellation	8
2.4.2	Further Examples	10
	References	15
3	Quadrature	17
3.1	Setting of the Problem and Examples	17
3.1.1	Quadrature Formulae	17
3.1.2	Interpolatory Quadrature	18
3.1.3	Newton–Cotes Quadrature	19
3.1.4	Gauss Quadrature	19
3.2	Consistency	19
3.3	Convergence	22
3.3.1	Definitions and Estimates	22
3.3.2	Functionals, Dual Norm, and Dual Space	23
3.4	Stability	25
3.4.1	Amplification of the Input Error	25
3.4.2	Definition of Stability	25
3.4.3	Stability of Particular Quadrature Formulae	26
3.4.4	Romberg Quadrature	28
3.4.5	Approximation Theorem of Weierstrass	30
3.4.6	Convergence Theorem	35

3.4.7	Uniform Boundedness Theorem	36
3.4.8	Necessity of the Stability Condition, Equivalence Theorem .	38
3.4.9	Modified Definitions for Consistency and Convergence	39
3.5	Further Remarks	41
3.5.1	General Intervals and Product Quadrature	41
3.5.2	Consistency Versus Stability	42
3.5.3	Perturbations	42
3.5.4	Arbitrary Slow Convergence Versus Quantitative Convergence	43
	References	45
4	Interpolation	47
4.1	Interpolation Problem	47
4.2	Convergence and Consistency	49
4.3	Stability	49
4.4	Equivalence Theorem	51
4.5	Instability of Polynomial Interpolation	51
4.6	Is Stability Important for Practical Computations?	53
4.7	Tensor Product Interpolation	55
4.8	Stability of Piecewise Polynomial Interpolation	56
4.8.1	Case of Local Support	56
4.8.2	Spline Interpolation as an Example for Global Support	57
4.9	From Point-wise Convergence to Operator-Norm Convergence	59
4.10	Approximation	60
	References	62
5	Ordinary Differential Equations	63
5.1	Initial-Value Problem	63
5.1.1	Setting of the Problem	63
5.1.2	One-Step Methods	64
5.1.3	Multistep Methods	65
5.2	Fixed-Point Theorem and Recursive Inequalities	66
5.3	Well-Conditioning of the Initial-Value Problem	68
5.4	Analysis of One-Step Methods	70
5.4.1	Implicit Methods	70
5.4.2	Lipschitz Continuity of ϕ	71
5.4.3	Consistency	71
5.4.4	Convergence	72
5.4.5	Stability	72
5.5	Analysis of Multistep Methods	74
5.5.1	Local Discretisation Error, Consistency	75
5.5.2	Convergence	75
5.5.3	Stability	76
5.5.4	Difference Equations	76
5.5.5	Stability and Convergence Theorems	83

5.5.6	Construction of Optimal Multistep Methods	85
5.5.7	Further Remarks	91
5.5.8	Other Stability Concepts	92
References		92
6	Instationary Partial Differential Equations	93
6.1	Introduction and Examples	93
6.1.1	Notation, Problem Setting, Function Spaces	93
6.1.2	The Hyperbolic Case $A = a\partial/\partial x$	95
6.1.3	The Parabolic Case $A = \partial^2/\partial x^2$	96
6.2	Semigroup of Solution Operators	98
6.3	Discretisation of the Partial Differential Equation	100
6.3.1	Notations	100
6.3.2	Transfer Operators r, p	101
6.3.3	Difference Schemes	102
6.4	Consistency, Convergence, and Stability	105
6.4.1	Definitions	105
6.4.2	Convergence, Stability and Equivalence Theorems	106
6.4.3	Other Norms	108
6.5	Sufficient and Necessary Conditions for Stability	109
6.5.1	First Criteria	109
6.5.2	Fourier Analysis	115
6.5.3	Further Criteria	118
6.5.4	Implicit Schemes	121
6.5.5	Vector-Valued Grid Functions	124
6.5.6	Generalisations	129
6.5.7	Dissipativity for Parabolic Discretisations	135
6.6	Consistency Versus Stability	135
References		137
7	Stability for Discretisations of Elliptic Problems	139
7.1	Elliptic Differential Equations	139
7.2	Discretisation	140
7.3	General Concept	142
7.3.1	Consistency	142
7.3.2	Convergence	143
7.3.3	Stability	144
7.4	Application to Difference Schemes	145
7.4.1	Classical Choice of Norms	145
7.4.2	Bijectivity of L	147
7.5	Finite Element Discretisation	149
7.5.1	Variational Problem	149
7.5.2	Galerkin Discretisation	150
7.5.3	Consistency	151
7.5.4	Convergence and Stability	153

7.5.5	Quantitative Discretisation Error and Regularity	153
7.5.6	L^2 Error	154
7.5.7	Stability of Saddle Point Problems	155
7.5.8	Further Remarks	157
7.5.9	Consistency Versus Stability	163
	References	165
8	Stability for Discretisations of Integral Equations	167
8.1	Integral Equations and Their Discretisations	167
8.1.1	Integral Equation, Banach Space	167
8.1.2	Discretisations	169
8.2	Stability Theory	172
8.2.1	Consistency	172
8.2.2	Stability	172
8.2.3	Convergence	173
8.2.4	Equivalence	174
8.3	Projection Methods	175
8.4	Stability Theory for Nyström's Method	176
8.5	Perturbation Results	180
8.6	Application to Eigenvalue Problems	181
	References	184
	Index	185