Preface

Logic appears in a 'sacred' and in a 'profane' form; the sacred form is dominant in proof theory, the profane form in model theory. The phenomenon is not unfamiliar, one observes this dichotomy also in other areas, e.g. set theory and recursion theory. Some early catastrophies, such as the discovery of the set theoretical paradoxes (Cantor, Russell), or the definability paradoxes (Richard, Berry), make us treat a subject for some time with the utmost awe and diffidence. Sooner or later, however, people start to treat the matter in a more free and easy way. Being raised in the 'sacred' tradition, my first encounter with the profane tradition was something like a culture shock. Hartley Rogers introduced me to a more relaxed world of logic by his example of teaching recursion theory to mathematicians as if it were just an ordinary course in, say, linear algebra or algebraic topology. In the course of time I have come to accept this viewpoint as the didactically sound one: before going into esoteric niceties one should develop a certain feeling for the subject and obtain a reasonable amount of plain working knowledge. For this reason this introductory text sets out in the profane vein and tends towards the sacred only at the end.

The present book has developed out of courses given at the mathematics department of Utrecht University. The experience drawn from these courses and the reaction of the participants suggested strongly that one should not practice and teach logic in isolation. As soon as possible examples from everyday mathematics should be introduced; indeed, first-order logic finds a rich field of applications in the study of groups, rings, partially ordered sets, etc.

The role of logic in mathematics and computer science is two-fold — a tool for applications in both areas, and a technique for laying the foundations. The latter role will be neglected here, we will concentrate on the daily matters of formalised (or formalizable) science. Indeed, I have opted for a practical approach, — I will cover the basics of proof techniques and semantics, and then go on to topics that are less abstract. Experience has taught us that the natural deduction technique of Gentzen lends itself best to an introduction, it is close enough to actual informal reasoning to enable students to devise proofs by themselves. Hardly any artificial tricks are involved and at the end there is the pleasing discovery that the system has striking structural properties, in particular it perfectly suits the constructive interpretation of logic and it allows normal forms. The latter topic has been added to this edition in view of its importance in theoretical computer science. In chapter 3 we already have enough technical power to obtain some of the traditional and (even today) surprising model theoretic results.

The book is written for beginners without knowledge of more advanced topics, no esoteric set theory or recursion theory is required. The basic ingredients are natural deduction and semantics, the latter is presented in constructive and classical form.

In chapter 5 intuitionistic logic is treated on the basis of natural deduction without the rule of Reductio ad absurdum, and of Kripke semantics. Intuitionistic logic has gradually freed itself from the image of eccentricity and now it is recognised for its usefulness in e.g., topos theory and type theory, hence its inclusion in a introductory text is fully justified. The final chapter, on normalisation, has been added for the same reasons; normalisation plays an important role in certain parts of computer science; traditionally normalisation (and cut elimination) belong to proof theory, but gradually applications in other areas have been introduced. In chapter 6 we consider only weak normalisation, a number of easy applications is given.

Various people have contributed to the shaping of the text at one time or another; Dana Scott, Jane Bridge, Henk Barendregt and Jeff Zucker have been most helpful for the preparation of the first edition. Since then many colleagues and students have spotted mistakes and suggested improvements; this edition benefited from the remarks of Eleanor McDonnell, A. Scedrov and Karst Koymans. To all of these critics and advisers I am grateful.

Progress has dictated that the traditional typewriter should be replaced by more modern devices; this book has been redone in LaTeX by Addie Dekker and my wife Doke. Addie led the way with the first three sections of chapter one and Doke finished the rest of the manuscript; I am indebted to both of them, especially to Doke who found time and courage to master the secrets of the LaTeX trade. Thanks go to Leen Kievit for putting together the derivations and for adding the finer touches required for a LaTeX manuscript. Paul Taylor's macro for proof trees has been used for the natural deduction derivations.

June 1994 Dirk van Dalen

The conversion to TeX has introduced a number of typos that are corrected in the present new printing. Many readers have been so kind to send me their collection of misprints, I am grateful to them for their help. In particular I want to thank Jan Smith, Vincenzo Scianna, A. Ursini, Mohammad Ardeshir, and Norihiro Kamide. Here in Utrecht my logic classes have been very helpful; in particular Marko Hollenberg, who taught part of a course, has provided me

with useful comments. Thanks go to them too.
I have used the occasion to incorporate a few improvements. The definition of 'subformula' has been streamlined – together with the notion of positive and negative occurrence. There is also a small addendum on 'induction on the rank of a formula'.

January 1997 Dirk van Dalen

At the request of users I have added a chapter on the incompleteness of arithmetic. It makes the book more self-contained, and adds useful information on basic recursion theory and arithmetic. The coding of formal arithmetic makes use of the exponential; this is not the most efficient coding, but for the heart of the argument that is not of the utmost importance. In order to avoid extra work the formal system of arithmetic contains the exponential. As the proof technique of the book is that of natural deduction, the coding of the notion of derivability is also based on it. There are of course many other approaches. The reader is encouraged to consult the literature.

The material of this chapter is by and large that of a course given in Utrecht in 1993. Students have been most helpful in commenting on the presentation, and in preparing TEX versions. W. Dean has kindly pointed out some more corrections in the old text.

The final text has benefited from comments and criticism of a number of colleagues and students. I am grateful for the advice of Lev Beklemishev, John Kuiper, Craig Smoryński, and Albert Visser. Thanks are due to Xander Schrijen, whose valuable assistance helped to overcome the TEX-problems.

May 2003 Dirk van Dalen

A number of corrections has been provided by Tony Hurkens; furthermore, I am indebted to him and Harold Hodes for pointing out that the definition of "free for" was in need of improvement. Sjoerd Zwart found a nasty typo that had escaped me and all (or most) readers.

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