## Contents

## **Positivity in Natural Sciences**

Jac	ek Be	anasiak	1
1	Introduction		
	1.1	What can go Wrong?	3
	1.2	And if Everything Seems to be Fine?	3
2	Spectral Properties of Operators		
	2.1	Operators	5
	2.2	Spectral Properties of a Single Operator	7
3	Banach Lattices and Positive Operators		
	3.1	Defining Order	13
	3.2	Banach Lattices	15
	3.3	Positive Operators	19
	3.4	Relation Between Order and Norm	20
	3.5	Complexification	23
	3.6	Spectral Radius of Positive Operators	24
4	First Semigroups		
	4.1	Around the Hille–Yosida Theorem	27
	4.2	Dissipative Operators	28
	4.3	Long Time Behaviour of Semigroups	29
	4.4	Positive Semigroups	37
	4.5	Generation Through Perturbation	39
	4.6	Positive Perturbations of Positive Semigroups	42
5	What can go Wrong?		
	5.1	Applications to Birth-and-Death Type Problems	52
	5.2	Chaos in Population Theory	59
6	Asynchronous Growth		61
	6.1	Essential Growth Bound	61
	6.2	Peripheral Spectrum of Positive Semigroups	63
	6.3	Compactness, Positivity and Irreducibility of Perturbed	
		Semigroups	67



References       87         Rescaling Stochastic Processes: Asymptotics       91         1 Introduction       91         1.1 First Examples of Rescaling       95         2 Stochastic Processes       97         2.1 Processes with Independent Increments       100         2.2 Martingales       100         2.3 Markov Processes       103         2.4 Brownian Motion and the Wiener Process       109         3 Itô Calculus       110         3.1 The Itô Integral       110         3.2 The Stochastic Differential       112         3.3 Stochastic Differential Equations       113         3.4 Kolmogorov and Fokker-Planck Equations       115         3.5 The Multidimensional Case       117         4 Deterministic Approximation of Stochastic Systems       118         4.1 Continuous Approximation of Stochastic Interacting Particle Systems       120         4.3 Convergence of the Empirical Measure       122         5 A Specific Model for Interacting Particles       128         5.1 Asymptotic Behavior of the System for Large Populations: A Heuristic Derivation       130         5.2 Asymptotic Behavior of the System for Large Populations: A Rigorous Derivation       130         5.2 Asymptotic Behavior of the System for Large Populations: A Rigorous Derivation
Rescaling Stochastic Processes: Asymptotics       91         1       Introduction       91         1.1       First Examples of Rescaling       95         2       Stochastic Processes       97         2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       110         3.3       Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting Particle Systems       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       <
Rescaling Stochastic Processes: Asymptotics       91         1       Introduction       91         1.1       First Examples of Rescaling       95         2       Stochastic Processes       97         2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting Particle Systems       120         4.3       Convergence of the Empirical Measure       120         4.3       Convergence of the Empirical Measure       120         5.1       Asymptotic Behavior of the System for Large Populations: A Rigorous Derivation       130         5.2       Asymptotic
V. Capasso and D. Morale       91         1       Introduction       91         1.1       First Examples of Rescaling       95         2       Stochastic Processes       97         2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting Particle Systems       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations: A Heuristic Derivation       130         5.2       Asymptotic Behavior of t
1       Introduction       91         1.1       First Examples of Rescaling       95         2       Stochastic Processes       97         2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5.4       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification
1.1       First Examples of Rescaling       95         2       Stochastic Processes       97         2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7
2       Stochastic Processes       97         2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification of the Limit $\rho$ 134
2.1       Processes with Independent Increments       100         2.2       Martingales       100         2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       110         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification of the Limit $\rho$ 134
2.2Martingales1002.3Markov Processes1032.4Brownian Motion and the Wiener Process1093Itô Calculus1103.1The Itô Integral1103.2The Stochastic Differential1123.3Stochastic Differential Equations1133.4Kolmogorov and Fokker-Planck Equations1153.5The Multidimensional Case1174Deterministic Approximation of Stochastic Systems1184.1Continuous Approximation of Stochastic Interacting Particle Systems1204.3Convergence of the Empirical Measure1225A Specific Model for Interacting Particles1285.1Asymptotic Behavior of the System for Large Populations: A Heuristic Derivation1305.2Asymptotic Behavior of the System for Large Populations: A Rigorous Derivation1346Long Time Behavior: Invariant Measure137AProof of the Identification of the Limit $\rho$ 141
2.3       Markov Processes       103         2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Jump Population Processes       118         4.2       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification of the Limit $\rho$ 141
2.4       Brownian Motion and the Wiener Process       109         3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification of the Limit $\rho$ 137
3       Itô Calculus       110         3.1       The Itô Integral       110         3.2       The Stochastic Differential Equations       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       130         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification of the Limit $\rho$ 141
3.1       The Itô Integral       110         3.2       The Stochastic Differential Equations       112         3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Jump Population Processes       118         4.2       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         7       Proof of the Identification of the Limit $\rho$ 141
3.2       The Stochastic Differential
3.3       Stochastic Differential Equations       113         3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Jump Population Processes       118         4.2       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         A       Proof of the Identification of the Limit $\rho$ 141
3.4       Kolmogorov and Fokker-Planck Equations       115         3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Jump Population Processes       118         4.2       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         A       Proof of the Identification of the Limit $\rho$ 141
3.5       The Multidimensional Case       117         4       Deterministic Approximation of Stochastic Systems       118         4.1       Continuous Approximation of Jump Population Processes       118         4.2       Continuous Approximation of Stochastic Interacting       120         4.3       Convergence of the Empirical Measure       122         5       A Specific Model for Interacting Particles       128         5.1       Asymptotic Behavior of the System for Large Populations:       130         5.2       Asymptotic Behavior of the System for Large Populations:       134         6       Long Time Behavior: Invariant Measure       137         A       Proof of the Identification of the Limit $\rho$ 141
4       Deterministic Approximation of Stochastic Systems
<ul> <li>4.1 Continuous Approximation of Jump Population Processes118</li> <li>4.2 Continuous Approximation of Stochastic Interacting Particle Systems</li></ul>
<ul> <li>4.2 Continuous Approximation of Stochastic Interacting Particle Systems</li></ul>
Particle Systems       120         4.3 Convergence of the Empirical Measure       122         5 A Specific Model for Interacting Particles       128         5.1 Asymptotic Behavior of the System for Large Populations:       130         5.2 Asymptotic Behavior of the System for Large Populations:       130         6 Long Time Behavior: Invariant Measure       137         A Proof of the Identification of the Limit ρ       141
<ul> <li>4.3 Convergence of the Empirical Measure</li></ul>
<ul> <li>5 A Specific Model for Interacting Particles</li></ul>
<ul> <li>5.1 Asymptotic Behavior of the System for Large Populations: A Heuristic Derivation</li></ul>
A Heuristic Derivation       130         5.2 Asymptotic Behavior of the System for Large Populations:       134         6 Long Time Behavior: Invariant Measure       137         A Proof of the Identification of the Limit ρ       141
<ul> <li>5.2 Asymptotic Behavior of the System for Large Populations: A Rigorous Derivation</li></ul>
A Rigorous Derivation       134         6 Long Time Behavior: Invariant Measure       137         A Proof of the Identification of the Limit $\rho$ 141
6 Long Time Behavior: Invariant Measure
A Proof of the Identification of the Limit $\rho$
References
Modelling Aspects of Concer Crowthy Insight
from Mothematical and Numerical Analysis
and Computational Simulation
Mark A. I. Charlein 147
1 Introduction 147
1 Introduction
1.1 Macroscopic Modelling
1.2     Cancer Growth and Development     149       2     Modelling Ausseular Solid Tumour Crowth     150
2 Modeling Avascular Solid Tullour Growth
2.1 Introduction
2.2 Interface Stability Theory
and Invasion

	2.4	Model Extension: Application to a Growing			
		Spherical Tumour			
	2.5	Discussion and Conclusions157			
3	Matl	Mathematical Modelling of T-Lymphocyte Response			
	to a Solid Tumour				
	3.1	Introduction			
	3.2	The Mathematical Model			
	3.3	Travelling Wave Analysis			
	3.4	Discussion			
4	Matl	hematical Modelling of Cancer Invasion			
	4.1	Introduction			
	4.2	Cancer Invasion of Tissue and Metastasis			
	4.3	Proteolysis and Extracellular Matrix Degradation			
	4.4	The Mathematical Model of Proteolysis and Cancer Cell			
		Invasion of Tissue			
	4.5	Nondimensionalisation of the Model Equations			
	4.6	Model Analysis			
	4.7	Spatially Uniform Steady States			
	4.8	Taxis-Driven Instability and Dispersion Curves			
	4.9	Numerical Results			
	4.10	Numerical Technique			
	4.11	Computational Simulation Results			
	4.12	Discussion and Conclusions191			
<b>5</b>	Sum	mary			
Ref	erenc	es			
Tin	s Po	twoon Microscopic and Macroscopic Descriptions			
Mir		Lash survey 201			
1	Intra	Duchowicz			
1	Mion	ouucion			
2 2	Can	oscopic (Stochastic) Systems			
ა 4	Gene	Praized Kinetic Models			
4 E		sive Limit			
о с	Links in the Space-Homogeneous Case				
07	Coag	Sunation-Fragmentation Equations			
1	Ine D	Space-Innomogeneous Case: Reaction-Diffusion Equations245			
8	Reac	tion–Diffusion–Chemotaxis Equations			
Rep	erenc	es			
Eve	olutio	onary Game Theory and Population Dynamics			
Jac	ek M	iękisz			
1	Shor	t Overview			
2	Intro	oduction			
3	A Cı	rash Course in Game Theory			
4	Repl	icator Dynamics			
5	Repl	icator Dynamics with Migration			

6	Rep	licator Dynamics with Time Delay		
	6.1	Social-Type Time Delay		
	6.2	Biological-Type Time Delay		
7	Stoc	hastic Dynamics of Finite Populations		
8	Stoc	hastic Dynamics of Well-Mixed Populations		
9	Spatial Games with Local Interactions			
	9.1	Nash Configurations and Stochastic Dynamics		
	9.2	Ground States and Nash Configurations		
	9.3	Ensemble Stability		
	9.4	Stochastic Stability in Non-Potential Games		
	9.5	Dominated Strategies		
10	Revi	ew of Other Results		
Ref	erenc	es		
List	t of ]	Participants		
Ind	ex .			