

Contents

1 Overview of Mathematical Methods in Partial Differential Equations	1
1.1 Comparison Principles	1
1.2 Radial Symmetry of Solutions to Semilinear Elliptic Equations	6
1.3 Variational Methods	9
1.3.1 Ekeland's Variational Principle	9
1.3.2 Mountain Pass Theorem	11
1.3.3 Around the Palais–Smale Condition for Even Functionals	12
1.3.4 Bolle's Variational Method for Broken Symmetries	14
1.4 Degree Theory	15
1.4.1 Brouwer Degree	15
1.4.2 Leray–Schauder Degree	16
1.4.3 Leray–Schauder Degree for Isolated Solutions	17
2 Liouville Type Theorems for Elliptic Operators in Divergence Form	19
2.1 Introduction	19
2.2 Some Related ODE Problems	21
2.3 Main Results	26
3 Blow-Up Boundary Solutions of the Logistic Equation	29
3.1 Singular Solutions of the Logistic Equation	30
3.1.1 A Karamata Regular Variation Theory Approach	43

3.2	Keller–Osserman Condition Revisited	60
3.2.1	Setting of the Problem	61
3.2.2	Minimality Principle	65
3.2.3	Existence of Solutions on Some Ball	70
3.2.4	Existence of Solutions on Small Balls	72
3.2.5	Existence of Solutions on Smooth Domains	73
3.2.6	Blow-Up Rate of Radially Symmetric Solutions	74
3.2.7	Blow-Up Rate of Solutions on Smooth Domains	75
3.2.8	A Uniqueness Result	77
3.2.9	Discrete Equations	79
3.2.10	Numerical Computations	86
3.3	Entire Large Solutions	91
3.3.1	A Useful Result: Bounded Entire Solutions	91
3.3.2	Existence of an Entire Large Solution	93
3.3.3	Uniqueness of Solution	98
3.4	Elliptic Equations with Absorption	100
3.5	Lack of the Keller–Osserman Condition	106
4	Singular Lane–Emden–Fowler Equations and Systems	117
4.1	Bifurcation Problems for Singular Elliptic Equations	117
4.2	Lane–Emden–Fowler Systems with Negative Exponents	130
4.2.1	Preliminary Results	132
4.2.2	Nonexistence of a Solution	139
4.2.3	Existence of a Solution	142
4.2.4	Regularity of Solution	149
4.2.5	Uniqueness	153
4.3	Sublinear Lane–Emden Systems with Singular Data	155
4.3.1	Case $p > 0$ and $q > 0$	155
4.3.2	Case $p > 0$ and $q < 0$	158
4.3.3	Case $p < 0$ and $q < 0$	162
4.3.4	Further Extensions: Superlinear Case	163
5	Singular Elliptic Inequalities in Exterior Domains	167
5.1	Introduction	167
5.2	Some Elliptic Problems in Bounded Domains	168

5.3	An Equivalent Integral Condition	174
5.4	The Nondegenerate Case	175
5.4.1	Nonexistence Results	175
5.4.2	Existence Results	180
5.5	The Degenerate Case	188
5.6	Application to Singular Elliptic Systems in Exterior Domains	203
6	Two Quasilinear Elliptic Problems	211
6.1	A Degenerate Elliptic Problem with Lack of Compactness	211
6.1.1	Introduction	211
6.1.2	Auxiliary Results	214
6.1.3	Proof of the Main Result	222
6.2	A Quasilinear Elliptic Problem for p -Laplace Operator	227
7	Some Classes of Polyharmonic Problems	245
7.1	An Eigenvalue Problem with Continuous Spectrum	245
7.2	Infinitely Many Solutions for Perturbed Nonlinearities	251
7.3	A Biharmonic Problem with Singular Nonlinearity	258
8	Large Time Behavior of Solutions for Degenerate Parabolic Equations	267
8.1	Introduction	267
8.2	Superlinear Case	268
8.3	Sublinear Case	275
8.4	Linear Case	283
9	Reaction-Diffusion Systems Arising in Chemistry	287
9.1	Introduction	287
9.2	Brusselator Model	288
9.2.1	Existence of Global Solutions	290
9.2.2	Stability of the Uniform Steady State	293
9.2.3	Diffusion-Driven Instability	295
9.2.4	A Priori Estimates	296
9.2.5	Nonexistence Results	299
9.2.6	Existence Results	302

9.3	Schnakenberg Model	306
9.3.1	The Evolution System and Global Solutions	307
9.3.2	A Priori Estimates	310
9.3.3	Nonexistence of Nonconstant Steady States	314
9.3.4	Existence Results	317
9.4	Lengyel–Epstein Model	322
9.4.1	Global Solutions in Time	323
9.4.2	Turing Instabilities	326
9.4.3	A Priori Estimates for Stationary Solutions	328
9.4.4	Nonexistence Results	330
9.4.5	Existence	332
10	Pattern Formation and the Gierer–Meinhardt Model in Molecular Biology	337
10.1	Introduction	337
10.2	Some Preliminaries	340
10.3	Case $0 \leq p < 1$	347
10.3.1	Existence	347
10.3.2	Further Results on Regularity	354
10.3.3	Uniqueness of a Solution	356
10.4	Case $p < 0$	362
10.4.1	A Nonexistence Result	362
10.4.2	Existence	364
A	Caffarelli–Kohn–Nirenberg Inequality	369
B	Estimates for the Green Function Associated to the Biharmonic Operator	373
References	377
Index	387