

Contents

1	Introduction and Philosophy	15
2	The Solute Transfer Function for Transport through Soil	21
2.1	The Solute Transport Volume	21
2.2	The Travel Time PDF	24
	Excursus: The Delta Function $\delta(t)$ and the Heaviside Function $H(t)$	25
2.3	Measurement of the Travel Time PDF $f^f(\ell, t)$	27
2.4	Measurement of the Travel Time CDF $P^f(\ell, t)$	28
	Excursus: Properties of Probability Distributions	28
2.5	Transfer Function Representation of Process Models	30
	Example 2.1: The Piston Flow Model	30
	Example 2.2: The Convection-Dispersion Equation (CDE)	31
	Example 2.3: Travel Time Moments of the Piston Flow Model	34
	Example 2.4: Travel Time Moments of the CDE	34
2.6	Process Assumptions and Travel Time PDFs	35
2.7	Stochastic-Convective and Convective-Dispersive Models	42
3	Flux and Resident Concentrations	45
3.1	Types of Solute Concentration	45
3.2	The Solute Conservation Equation	46
	Example 3.1: Resident Concentrations of the CDE	47
	Example 3.2: The Mobile-Immobile Water Model (MIM)	50
	Example 3.3: Travel Time Moments of the MIM	51
3.3	Flux and Resident PDFs	53
3.4	Depth Moments of the Resident PDF	54
	Example 3.4: Depth Moments of the CDE	55
3.5	Resident PDF of the Stochastic-Convective Model	56

Example 3.5: Depth Moments of the Stochastic-Convective Model	57
3.6 The Initial Value Problem	58
3.6.1 The Initial Value Problem in Infinite Soil	58
Example 3.6: The Resident PDF of the CDE in Infinite Soil	58
3.6.2 The Initial Value Problem of the Semi-Infinite CDE Model	59
3.6.3 The Initial Value Problem of the Stochastic-Convective Model	59
4 Stochastic Stream Tube Modeling	63
Example 4.1: Piston Flow Through Parallel Soil Columns	64
Example 4.2: The Random Water Content Model	65
4.1 Solute Transport and Adsorption	66
4.2 Linear Equilibrium Adsorption	66
Example 4.3: Linear Adsorption and the CDE	67
4.3 Rate Limited Adsorption	68
Example 4.4: Rate Limited Adsorption and the CDE	68
4.4 Spatial Variability and Equilibrium Adsorption	70
Example 4.5: Spatial Variability of Pesticide Adsorption	72
4.5 Moments of Stream Tube Models	73
Example 4.6: Moments of the Parallel Soil Column Model	74
4.6 Stream Tube Modeling and Variability Analysis	75
Example 4.7: Solute Transport Under Spatially Variable Irrigation	75
4.7 Resident PDFs of Stream Tube Models	75
Example 4.8: Solute Movement in a Lognormal Velocity Field	77
4.8 Solute Transport with first order Decay	77
4.9 Simultaneous Transport, Adsorption, and first order Decay	79
4.9.1 Estimation of the Mean Residual Mass Fraction (RMF)	79
Example 4.9: Spatial Variability and Pesticide Ground Water Pollution Potential	80
4.9.2 Estimation of the Variance of the RMF	82
5 Transfer Functions for Vertically Heterogeneous Soils	85
5.1 Depth-Dependent Water Content	85
Example 5.1: Steady State Convective-Dispersive Flow in Heterogeneous Soil	86
5.2 Solute Transport in Layered Soils	88
Excursus: Sums and Products of Random Variables	89
5.2.1 Independent Soil Layers	91

5.2.2	Perfectly Correlated Soil Layers	91
	Example 5.3: The Zero-Correlation Convective-Dispersive Model	93
	Example 5.4: The Perfect Correlation Stochastic-Convective Model	94
5.3	Flux PDFs in Layered Soils	94
5.4	Convective-Dispersive Resident PDFs for Layered Soil	95
5.5	Stochastic-Convective Resident PDFs for Layered Soil	99
5.6	Solute Transport Under Depth-Dependent Adsorption	101
	Example 5.5: Depth-Dependent Adsorption	101
6	Practical Field Application of the Transfer Function Approach in Unsaturated Soil	105
6.1	A Transient Transfer Function Model	106
6.1.1	The Unique Drainage PDF	107
6.1.2	Stochastic-Convective Flow	108
6.1.3	Transient Convolution Integral	108
6.1.4	Adjustment for Transient Water Flow	109
6.2	Water Balance Model	111
6.3	Stochastic Simulation of Transient Water Flow	112
	Example 6.1: Climatic Variability and Pesticide Leaching	115
6.4	Scaling Theory	116
6.4.1	Transport Models Based on Scaling Theory	117
	Example 6.2: The Scaling Model of Bresler and Dagan	118
6.4.2	Tests of Scaling Theory	120
7	Stochastic Continuum Modeling	123
7.1	The N Layer Travel Time PDF	123
	Example 7.1: Multivariate Normally Distributed Travel Times	124
	Excursus: Random Functions and Stochastic Processes	126
7.2	Travel Time Representation in Terms of Solute Velocity	130
7.3	Homogeneous Stochastic Medium	131
	Excursus: Arbitrary Input Function	131
7.4	Numerical Simulation of Transport Through a Stochastic Medium	140
7.4.1	Homogeneous Soil	141
7.4.2	Horizontally Inhomogeneous Soil	142
	Example 7.2: Double Porous Medium without Horizontal Dispersion	144

A	Integral Transforms	151
A.1	The Laplace Transform	151
Example A.1:	Analytic Evaluation of Laplace Transforms	152
A.1.1	Transforms of Derivatives and Integrals	152
A.1.2	Transformed Solution of Differential Equations	154
Example A.2:	Laplace Transform Solution of the Damped Harmonic Oscillator	154
Example A.3:	Laplace Transform Solution of the Heat Equation	155
A.1.3	Inverse Laplace Transformation	156
Example A.4:	Inversion of the Transform of the Flux PDF of the CDE	157
Example A.5:	Inversion of the Transform of the Flux CDF of the CDE	158
Example A.6:	Inversion of the Flux PDF of the CDE Under Rate Limited Adsorption	159
A.2	The Fourier Transform	160
Example A.7:	Fourier Transform of the Exponential Covariance Function	160
Example A.8:	Inversion of the Fourier Transform of the Resident PDF of the CDE in Infinite Soil	161
B	Useful Fortran Computer Programs	163
B.1	Numerical Inversion of Laplace Transforms	163
B.2	Error Function Evaluation	165
C	Table of Laplace Transforms	169
D	Solutions to Problems	173
D.1	Problems in Chapter 1	173
D.2	Problems in Chapter 2	175
D.3	Problems in Chapter 3	180
D.4	Problems in Chapter 4	189
D.5	Problems in Chapter 5	195
D.6	Problems in Chapter 6	201
D.7	Problems in Chapter 7	207