

Table of Contents

Preface	V
1. The basic equations of fluid dynamics	1
1.1 Introduction	1
1.2 Vector analysis	5
1.3 The total derivative and the transport theorem	9
1.4 Conservation of mass	12
1.5 Conservation of momentum	13
1.6 Conservation of energy	19
1.7 Thermodynamic aspects	22
1.8 Bernoulli's theorem	26
1.9 Kelvin's circulation theorem and potential flow	28
1.10 The Euler equations	32
1.11 The convection-diffusion equation	33
1.12 Conditions for incompressible flow	34
1.13 Turbulence	37
1.14 Stratified flow and free convection	43
1.15 Moving frame of reference	47
1.16 The shallow-water equations	48
2. Partial differential equations: analytic aspects	53
2.1 Introduction	53
2.2 Classification of partial differential equations	54
2.3 Boundary conditions	61
2.4 Maximum principles	66
2.5 Boundary layer theory	70
3. Finite volume and finite difference discretization on nonuniform grids	81
3.1 Introduction	81
3.2 An elliptic equation	82
3.3 A one-dimensional example	84
3.4 Vertex-centered discretization	88
3.5 Cell-centered discretization	94

3.6	Upwind discretization	96
3.7	Nonuniform grids in one dimension	99
4.	The stationary convection-diffusion equation	111
4.1	Introduction	111
4.2	Finite volume discretization of the stationary convection-diffusion equation in one dimension	113
4.3	Numerical experiments on locally refined one-dimensional grid	120
4.4	Schemes of positive type	122
4.5	Upwind discretization	126
4.6	Defect correction	129
4.7	Péclet-independent accuracy in two dimensions	133
4.8	More accurate discretization of the convection term	148
5.	The nonstationary convection-diffusion equation	163
5.1	Introduction	163
5.2	Example of instability	164
5.3	Stability definitions.	166
5.4	The discrete maximum principle	170
5.5	Fourier stability analysis	171
5.6	Principles of von Neumann stability analysis	174
5.7	Useful properties of the symbol	178
5.8	Derivation of von Neumann stability conditions	184
5.9	Numerical experiments	208
5.10	Strong stability	217
6.	The incompressible Navier-Stokes equations	227
6.1	Introduction	227
6.2	Equations of motion and boundary conditions	227
6.3	Spatial discretization on colocated grid	232
6.4	Spatial discretization on staggered grid	240
6.5	On the choice of boundary conditions	244
6.6	Temporal discretization on staggered grid	249
6.7	Temporal discretization on colocated grid	261
7.	Iterative methods	263
7.1	Introduction	263
7.2	Stationary iterative methods	264
7.3	Krylov subspace methods	270
7.4	Multigrid methods	285
7.5	Fast Poisson solvers	292
7.6	Iterative methods for the incompressible Navier-Stokes equations	293

8. The shallow-water equations	305
8.1 Introduction	305
8.2 The one-dimensional case	305
8.3 The two-dimensional case	323
9. Scalar conservation laws	339
9.1 Introduction	339
9.2 Godunov's order barrier theorem	339
9.3 Linear schemes	346
9.4 Scalar conservation laws	361
10. The Euler equations in one space dimension	397
10.1 Introduction	397
10.2 Analytic aspects	397
10.3 The approximate Riemann solver of Roe	414
10.4 The Osher scheme	425
10.5 Flux splitting schemes	436
10.6 Numerical stability	442
10.7 The Jameson-Schmidt-Turkel scheme	447
10.8 Higher order schemes	456
11. Discretization in general domains	467
11.1 Introduction	467
11.2 Three types of grid	467
11.3 Boundary-fitted grids	470
11.4 Basic geometric properties of grid cells	474
11.5 Introduction to tensor analysis	484
11.5.1 Invariance	485
11.5.2 The geometric quantities	490
11.5.3 Tensor calculus	498
11.5.4 The equations of motion in general coordinates	501
12. Numerical solution of the Euler equations in general domains	503
12.1 Introduction	503
12.2 Analytic aspects	503
12.3 Cell-centered finite volume discretization on boundary-fitted grids	511
12.4 Numerical boundary conditions	518
12.5 Temporal discretization	525
13. Numerical solution of the Navier-Stokes equations in general domains	531
13.1 Introduction	531
13.2 Analytic aspects	531

13.3 Colocated scheme for the compressible Navier-Stokes equations	533
13.4 Colocated scheme for the incompressible Navier-Stokes equations	535
13.5 Staggered scheme for the incompressible Navier-Stokes equations	538
13.6 An application	557
13.7 Verification and validation	559
14. Unified methods for computing incompressible and compressible flow	567
14.1 The need for unified methods	567
14.2 Difficulties with the zero Mach number limit	568
14.3 Preconditioning	571
14.4 Mach-uniform dimensionless Euler equations	578
14.5 A staggered scheme for fully compressible flow	583
14.6 Unified schemes for incompressible and compressible flow. ...	589
References	603
Index	633