

Contents

Braids and Knots	1
Patrick D. Bangert	
1 Physical Knots and Braids: A History and Overview	2
2 Braids and the Braid Group	4
2.1 The Topological Idea	4
2.2 The Origin of Braid Theory	5
2.3 The Topological Braid	9
2.4 The Braid Group	12
2.5 Other Presentations of the Braid Group	16
2.6 The Alexander and Jones Polynomials	18
2.7 Properties of the Braid Group	21
2.8 Algorithmic Problems in the Braid Groups	22
3 Braids and Knots	24
3.1 Notation for Knots	24
3.2 Braids to Knots	28
3.3 Example: The Torus Knots	28
3.4 Knots to Braids I: The Vogel Method	29
3.5 Knots to Braids II: An Axis for the Universal Polyhedron	31
3.6 Peripheral Group Systems of Closed Braids	39
4 Classification of Braids and Knots	45
4.1 The Word Problem I: Garside's Solution	45
4.2 The Word Problem II: Rewriting Systems	47
4.3 The Conjugacy Problem I: Garside's Solution	52
4.4 The Conjugacy Problem II: Rewriting Systems	54
4.5 Markov's Theorem	59
4.6 The Minimal Word Problem	62
5 Open Problems	69
References	70

Topological Quantities: Calculating Winding, Writhing, Linking, and Higher Order Invariants	75
Mitchell A. Berger (CIME Lecturer)	
1 Introduction	75
2 Winding Numbers	77
2.1 Two Braided Curves between Parallel Planes	77
2.2 General Curves	78
2.3 Topological Invariance	81
3 Linking Numbers	82
3.1 Winding Number Derivation	82
3.2 General Properties	83
4 Twist and Writhe Numbers	84
4.1 Ribbons	84
4.2 Twisted Tubes	86
5 Writhe from Winding Numbers	87
5.1 The Twist as a Function of Height	88
5.2 The Local Winding Number as a Function of Height	89
5.3 The Local Writhe as a Function of Height	90
5.4 The Nonlocal Winding Number as a Function of Height	91
5.5 Example: A Trefoil Torus Knot	92
6 Writhe for Open Curves	94
7 Higher Order Winding	96
References	97
Tangles, Rational Knots and DNA	99
Louis H. Kauffman (CIME Lecturer) and Sofia Lambropoulou	
1 Introduction	99
2 2-Tangles and Rational Tangles	102
3 Continued Fractions and the Classification of Rational Tangles	107
4 Alternate Definitions of the Tangle Fraction	111
4.1 $F(T)$ Through the Bracket Polynomial	111
4.2 The Fraction through Coloring	119
4.3 The Fraction through Conductance	121
5 The Classification of Unoriented Rational Knots	122
6 Rational Knots and Their Mirror Images	126
7 The Oriented Case	127
8 Strongly Invertible Links	131
9 Applications to the Topology of DNA	132
References	136
The Group and Hamiltonian Descriptions of Hydrodynamical Systems	139
Boris Khesin (CIME Lecturer)	
1 Introduction	139
2 Euler Equations and Geodesics	140
2.1 The Euler Equation of an Ideal Incompressible Fluid	140

2.2	Geodesics on Lie Groups	141
2.3	Geodesic Description for Various Equations	142
3	Euler Equations on Groups as Hamiltonian Systems and the Binormal Equation	142
3.1	Hamiltonian Reformulation of the Euler Equations	142
3.2	Hamiltonian Structure of the Landau-Lifschitz Equation	143
3.3	Properties of the Binormal Equation	145
4	The KdV-Type Equations as Euler Equations	147
4.1	The Virasoro Algebra and the KdV Equation	147
4.2	Similar Equations and Conservation Laws	149
5	Hamiltonian Structure of the Euler Equations for an Incompressible Fluid	150
5.1	The Euler Hydrodynamics as a Hamiltonian Equation	150
5.2	The Space of Knots and the Dual of the Lie Algebra of Divergence-Free Vector Fields	153
	References	154
Singularities in Fluid Dynamics and their Resolution		157
H.K. Moffatt (CIME Lecturer)		
1	Introduction	157
2	Boundary-Driven Singularities	158
3	Cusp Singularities at a Free Surface	160
4	A Simple Finite-Time Singularity: the Euler Disk	161
5	Finite-Time Singularities at Interior Points	162
	References	166
Structural Complexity and Dynamical Systems		167
Renzo L. Ricca (School Director and CIME Lecturer)		
1	Introduction	167
2	Helmholtz's Work on Vortex Motion: Birth of Topological Fluid Mechanics	168
2.1	Multi-Valued Potentials in Multiply Connected Regions	168
2.2	Green's Theorem in Multiply Connected Regions	172
2.3	Conservation Laws	172
3	Measures of Structural Complexity	173
3.1	Dynamical Systems and Vector Field Analysis	174
3.2	Measures of Tangle Complexity	175
4	Topological Bounds on Energy and Helicity-Crossing Number Relations for Magnetic Knots and Links	181
4.1	Topology Bounds Energy in Ideal Fluid	182
4.2	Helicity-Crossing Number Relations in Dissipative Fluid	184
	References	185

Random Knotting: Theorems, Simulations and Applications ..	187
De Witt Sumners (CIME Lecturer)	
1 Introduction	187
2 The Frischi-Wasserman-Delbruck Conjecture	189
3 Entanglement Complexity of Random Knots and Random Arcs	194
4 Writhe, Signature and Chirality of Random Knots	196
5 Application of Random Knotting to Viral DNA Packing	201
5.1 Knot Type Probabilities for P4 DNA in Free Solution	204
5.2 Monte Carlo Simulation	205
5.3 Results and Discussion Knot Complexity of DNA Molecules Extracted from Phage P4	206
5.4 Identification of Specific Knot Types by Their Location on the Gel	209
5.5 Monte Carlo Simulations of Random Knot Distributions in Confined Volumes	209
References	213
Index	219